MEDICAL EDUCATION RESOURCE

Measures of Central Tendency

Measures of Central Tendency

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1. Definition of Central Tendency

Core Definition

- A **measure of central tendency** is a **single value that represents the center of a dataset**
- It gives an idea about the **typical or average value**

Key Concepts

- **Summarization of data**
  - Converts large data into a **single representative value**
- **Representative value**
  - Reflects the **overall pattern of the dataset**

Easy Example

- Data: 2, 4, 6, 8, 10
- Central value ? around **6**
  ? This represents the dataset

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2. Types of Central Tendency ?

Mean (Arithmetic Mean)

Definition

- The **mean** is the **average of all observations**
- It is calculated by dividing the **sum of values by number of observations**

Formula ?

x?=?xn\bar{x} = \frac{\sum x}{n}x?=n?x?

For Grouped Data ?

x?=?fx?f\bar{x} = \frac{\sum f x}{\sum f}x?=?f?fx?

Easy Explanation

- Add all values ? divide by total number
- For grouped data ? multiply frequency with value

Example

- Data: 2, 4, 6, 8
- Mean = (2+4+6+8) / 4 = **5**

Important Exam Points ?

- Most commonly used measure
- Affected by **extreme values (outliers)**

Median

Definition

- **Median** is the **middle value** after arranging data in ascending or descending order

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Method of Calculation ?

Odd Number of Observations

- Median = value at position
  (n + 1) / 2

Even Number of Observations

- Median = average of two middle values
  n/2 and (n/2 + 1)

Worked Example ?

Odd Case

- Data: 1, 3, 5, 7, 9
- Median position = (5+1)/2 = 3rd value
- Median = **5**

Even Case

- Data: 2, 4, 6, 8
- Middle values = 4 and 6
- Median = (4+6)/2 = **5**

Easy Understanding

- Median divides data into **two equal halves**
- Not affected by **extreme values**

Important Exam Points ?

- Median = **middle value**
- Requires **arrangement of data**
- Preferred when data has **outliers**

Short Note (Revision)

- Mean ? average
- Median ? middle value
- Mean affected by outliers
- Median not affected

Mode

Definition

- **Mode** is the **most frequently occurring value** in a dataset
- It represents the value that appears **maximum number of times**

Easy Explanation

- Data: 2, 4, 4, 6, 8
- Mode = **4** (appears most frequently)

Characteristics ?

- **May not be unique**
  - Dataset can have:
    - One mode ? **Unimodal**
    - Two modes ? **Bimodal**
    - More ? **Multimodal**
- **Useful in categorical data**
  - Best for **qualitative data**
  - Example:
    - Most common blood group
    - Most common disease
- **Not affected by extreme values**
  - Outliers do not influence mode

Important Exam Points ?

- Mode = **most frequent value**
- Useful for **categorical data**
- May have **multiple modes**

Short Note (Revision)

- Most frequent value
- Can be multiple
- Useful in qualitative data

Characteristics of a Good Average ?

Essential Features

- **Simple to understand**
  - Should be easy for anyone to interpret
- **Easy to calculate**
  - Calculation should not be complicated
- **Based on all observations**
  - Should consider **entire dataset**
- **Not affected by extreme values**
  - Should be stable even if outliers are present
- **Capable of further analysis**
  - Should be useful for:
    - Statistical calculations
    - Comparisons
    - Research analysis

Easy Understanding

- A good average should be:
  - **Simple + reliable + representative**

Important Exam Point ?

- Ideal average = **simple, stable, representative, and usable for analysis**

Short Note (Revision)

- Simple
- Easy to calculate
- Uses all data
- Not affected by extremes
- Useful for analysis

Mean ?

Properties

- **Uses all data values**
  - Every observation contributes to the mean
- **Affected by extreme values (outliers)**
  - Very high or low values can **distort the mean**

Advantages

- **Mathematical usefulness**
  - Can be used for:
    - Further calculations (SD, variance, regression)
- **Stability**
  - Less fluctuation in repeated samples

Disadvantages

- **Influenced by outliers**
  - Not suitable for skewed data

Quick Example

- Data: 2, 4, 6, 8, 100
- Mean = **24** ? not representative due to outlier

Median ?

Properties

- **Not affected by extreme values**
  - Outliers do not change median
- **Divides data into two equal halves**
  - 50% values below, 50% above

Advantages

- **Suitable for skewed data**
  - Best when extreme values are present

Disadvantages

- **Does not use all observations**
  - Only depends on middle value

Quick Example

- Data: 2, 4, 6, 8, 100
- Median = **6** ? more representative

Mode

Properties

- **Most frequent value**
  - Highest occurrence in dataset

Advantages

- **Useful for nominal/categorical data**
  - Example:
    - Most common blood group
    - Most common disease

Disadvantages

- **May be multiple or no mode**
  - Data may be:
    - Bimodal
    - Multimodal
    - No mode

Quick Example

- Data: 2, 4, 4, 6, 8

- Mode = **4**

High-Yield Comparison (Exam Trick) ?

Mean

- Uses all data
- Affected by outliers
- Best for symmetrical data

Median

- Middle value
- Not affected by outliers
- Best for skewed data

Mode

- Most frequent value
- Used for categorical data
- May not be unique

Short Note (Revision)

- Mean ? average (affected by extremes)
- Median ? middle (stable)
- Mode ? most frequent

FEATURE	MEAN	MEDIAN	MODE
Definition	Sum of all values ÷ number of observations	Middle value after arrangement	Most frequent value
Data Used	Uses all observations	Does not use all values fully	Based on frequency
Effect of Outliers	Affected	Not affected	Not affected

FEATURE	MEAN	MEDIAN	MODE
Best Use	Symmetrical data, further calculations	Skewed data	Categorical/qualitative data
Example	Average marks	Income distribution	Most common blood group

Skewness ?

Definition

- **Skewness** is the **measure of asymmetry of a distribution**
- It shows whether data is **symmetrically distributed or shifted to one side**

Easy Explanation

- If data is evenly spread ? **Symmetrical**
- If tail extends to right/left ? **Skewed distribution**

Types of Skewness ?

1. Symmetrical Distribution

- Data is **evenly distributed on both sides**
- **Mean = Median = Mode**

2. Positive Skew (Right Skew)

- Tail extends towards **right side**
- Few **high extreme values present**
- Relationship:

- **Mean > Median > Mode**

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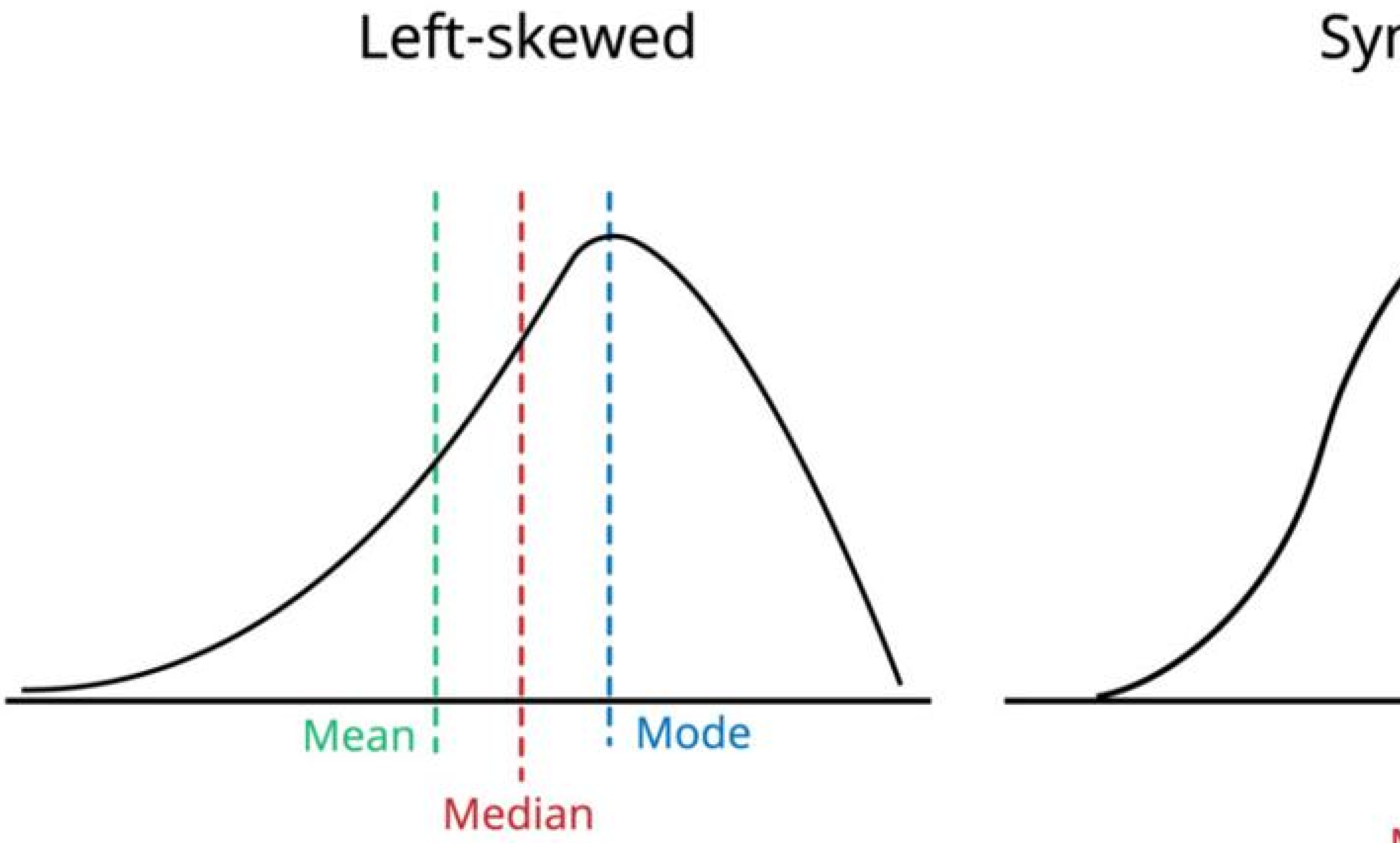
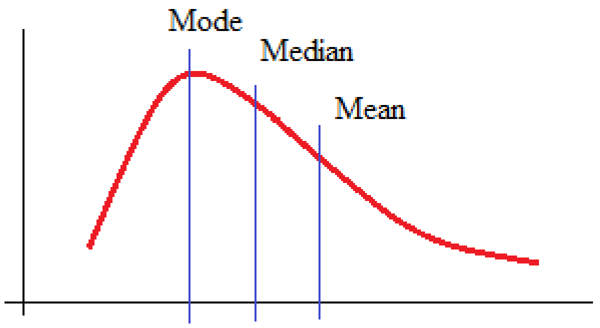
3. Negative Skew (Left Skew)

- Tail extends towards **left side**
- Few **low extreme values present**
- Relationship:
  - **Mean < Median < Mode**

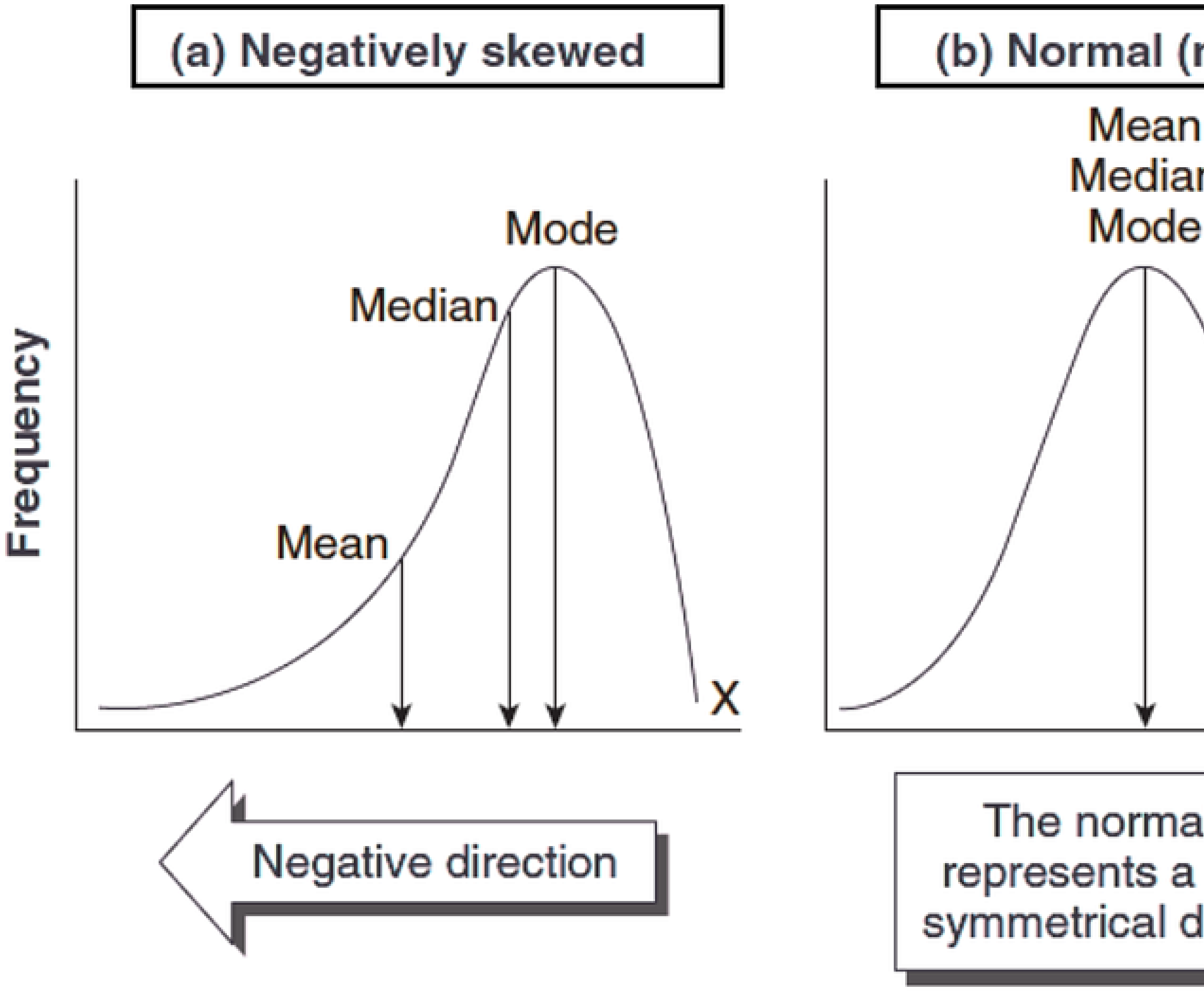
Diagrams (VERY IMPORTANT) ?


Left-skewed
Mean
Median
Mode
Sy



Positive-Skewness

Mean > Median > Mode


(a) Negatively skewed
Frequency
Mode
Median
Mean
X
Negative direction
(b) Normal (r
Mean
Median
Mode
The norma
represents a
symmetrical d

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Interpretation ?

Direction of Skewness

- **Right tail ? Positive skew**
- **Left tail ? Negative skew**

Clinical / Epidemiological Examples

- **Positive Skew**
    - Income distribution (few very high incomes)
    - Hospital stay duration (few long stays)
- **Negative Skew**
    - Age at death in developed countries (most live longer)
- **Symmetrical**
    - Normal distribution (e.g., height in population)

Important Exam Points ?

- Skewness = **asymmetry of distribution**
- Formulas to remember:
    - Positive skew ? Mean > Median > Mode
    - Negative skew ? Mean < Median < Mode

Short Note (Revision)

- Symmetrical ? Mean = Median = Mode
- Positive skew ? Right tail
- Negative skew ? Left tail

Measures of Dispersion

Definition of Dispersion

Core Definition

- **Dispersion** is the **measure of spread or variability of data**
- It shows how far the values are **scattered from the central value**

Easy Explanation

- Same mean, different spread:

  Data 1: 5, 5, 5, 5 ? **No dispersion**
  Data 2: 1, 5, 9, 5 ? **High dispersion**

Importance ?

- **Shows reliability**
  - Less dispersion ? data is more reliable
- **Indicates consistency**
  - Small spread ? consistent data
  - Large spread ? variable data

Types of Dispersion ?

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1. Range

Definition

- **Range** is the **difference between highest and lowest value**

Formula ?

Range=Max?Min\text{Range} = \text{Max} - \text{Min}Range=Max?Min

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Easy Example

- Data: 2, 4, 6, 8
- Range = 8 – 2 = **6**

Advantages

- Simple and easy to calculate
- Quick idea of spread

Limitations

- Uses only **two values (max & min)**
- Not reliable
- Affected by outliers

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2. Quartile Deviation (Semi-IQR)

Definition

- Based on **quartiles (Q1 and Q3)**
- Measures spread of **middle 50% data**

Formula ?

QD=Q3?Q12QD = \frac{Q_3 - Q_1}{2}QD=2Q3??Q1??

Easy Explanation

- Q1 ? 25th percentile
- Q3 ? 75th percentile
- Focuses on **central data**, ignores extremes

Advantages

- Not affected by extreme values
- Better than range

Limitations

- Does not use all data

- Limited mathematical use

3. Mean Deviation

Definition

- **Mean deviation** is the **average of absolute deviations from mean or median**

Easy Explanation

- Calculate how far each value is from mean
- Take average of those distances

Example (Concept)

- Data: 2, 4, 6
- Mean = 4
- Deviations: 2, 0, 2
- Mean deviation = (2+0+2)/3 = **1.33**

Advantages

- Uses all observations
- Better than range

Limitations

- Absolute values ? difficult for further calculations
- Less commonly used

Important Exam Points ?

- Range ? simplest
- QD ? middle spread
- Mean deviation ? average distance

Short Note (Revision)

- Dispersion = spread of data

- Range ? max – min
- QD ? (Q3 – Q1)/2
- Mean deviation ? average deviation

Standard Deviation (SD) ? MOST IMPORTANT

Definition

- **Standard deviation (SD)** is a measure of **variability of data around the mean**
- It tells how much the values **deviate (spread)** from the average

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Easy Explanation

- Small spread ? values close to mean ? **low SD**
- Large spread ? values far from mean ? **high SD**

Formula ?

For Individual Data

SD=?(x?x?)2nSD = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}SD=n?(x?x?)2??

For Grouped Data

SD=?f(x?x?)2?fSD = \sqrt{\frac{\sum f (x - \bar{x})^2}{\sum f}}SD=?f?f(x?x?)2??

Easy Steps (Exam Trick)

1. Find **mean (x?)**
2. Calculate **(x ? x?)**
3. Square ? **(x ? x?)²**
4. Take average
5. Take **square root**

Interpretation ?

- **Small SD**
    - Data is **closely clustered around mean**

    - More **consistent & reliable**
- **Large SD**
    - Data is **widely spread**
    - Less consistency

Example

- Data 1: 5, 5, 5, 5 ? SD = 0 (no variation)
- Data 2: 1, 5, 9, 5 ? SD is high

Important Exam Points ?

- Most important measure of dispersion
- Uses **all data values**
- Essential for:
    - Normal distribution
    - Z-score
    - Statistical tests

Short Note (Revision)

- SD = spread around mean
- Small SD ? consistent
- Large SD ? variable

Variance

Definition

- **Variance** is the **square of standard deviation**
- It measures spread in **squared units**

Formula ?

Variance=SD2\text{Variance} = SD^2Variance=SD2

Easy Explanation

- Variance = **average of squared deviations from mean**

- SD = ?Variance

Important Exam Points ?

- Variance = **SD²**
- Units are **squared**
- SD is preferred for interpretation

Short Note (Revision)

- Variance = square of SD
- SD more useful clinically

Properties of Standard Deviation ?

Key Properties

- **Always positive**
  - SD is **never negative**
  - Because deviations are **squared before calculation**
  - Minimum value = **0** (when all observations are same)

- **Based on all observations**
  - Every data value contributes to SD
  - Makes it a **reliable measure of dispersion**

- **Affected by extreme values (outliers)**
  - Very high or low values can **increase SD significantly**
  - Hence, SD is sensitive to skewed data

- **Algebraically tractable**
  - Can be used in **mathematical/statistical calculations**
  - Important for:
    - Variance
    - Z-score
    - Normal distribution

- Regression & correlation

Easy Understanding

- SD = **powerful** + **precise** + **mathematically useful**
- But ? **sensitive to outliers**

Important Exam Point ?

- SD is:
  - Always **positive**
  - Uses **all data**
  - **Affected by outliers**
  - **Mathematically useful**

Short Note (Revision)

- Always positive
- Uses all observations
- Affected by extremes
- Useful in calculations

Coefficient of Variation (CV) ?

Definition

- **Coefficient of Variation (CV)** is a **relative measure of variability**
- It expresses **standard deviation as a percentage of mean**
- Helps compare variability **between different datasets**

Formula ?

CV=SDx?×100CV = \frac{SD}{\bar{x}} \times 100CV=x?SD?×100

Easy Explanation

- CV tells how **large the variation is compared to the mean**
- Lower CV ? **more consistency**
- Higher CV ? **more variability**

Uses ?

- Compare **consistency between datasets**
- Used when:
  - Means are **different**
  - Units are **different**

Example (Comparison) ?

Dataset A

- Mean = 100
- SD = 10

CV = (10 / 100) × 100 = **10%**

Dataset B

- Mean = 50
- SD = 10

CV = (10 / 50) × 100 = **20%**

Interpretation ?

- Dataset A ? CV = 10% ? **More consistent**
- Dataset B ? CV = 20% ? **Less consistent (more variation)**

Exam Trick ?

- **Lower CV ? Better consistency**
- **Higher CV ? More variability**

Important Exam Points ?

- CV = **relative measure**

- Used for **comparison**
- Expressed in **percentage**

Short Note (Revision)

- CV = (SD/Mean) × 100
- Lower CV ? more stable
- Used to compare datasets

Normal Distribution & SD ?

Definition

- A **normal distribution** is a **symmetrical, bell-shaped distribution**
- Data is distributed evenly around the **mean**

Properties ?

- **Mean = Median = Mode**
  - All central tendencies coincide at the center
- **Symmetrical distribution**
  - Left side = Right side
- **Total area = 100%**
  - Entire curve represents **100% of data**

Easy Explanation

- Most values lie **near the mean**
- Few values lie at **extremes (tails)**

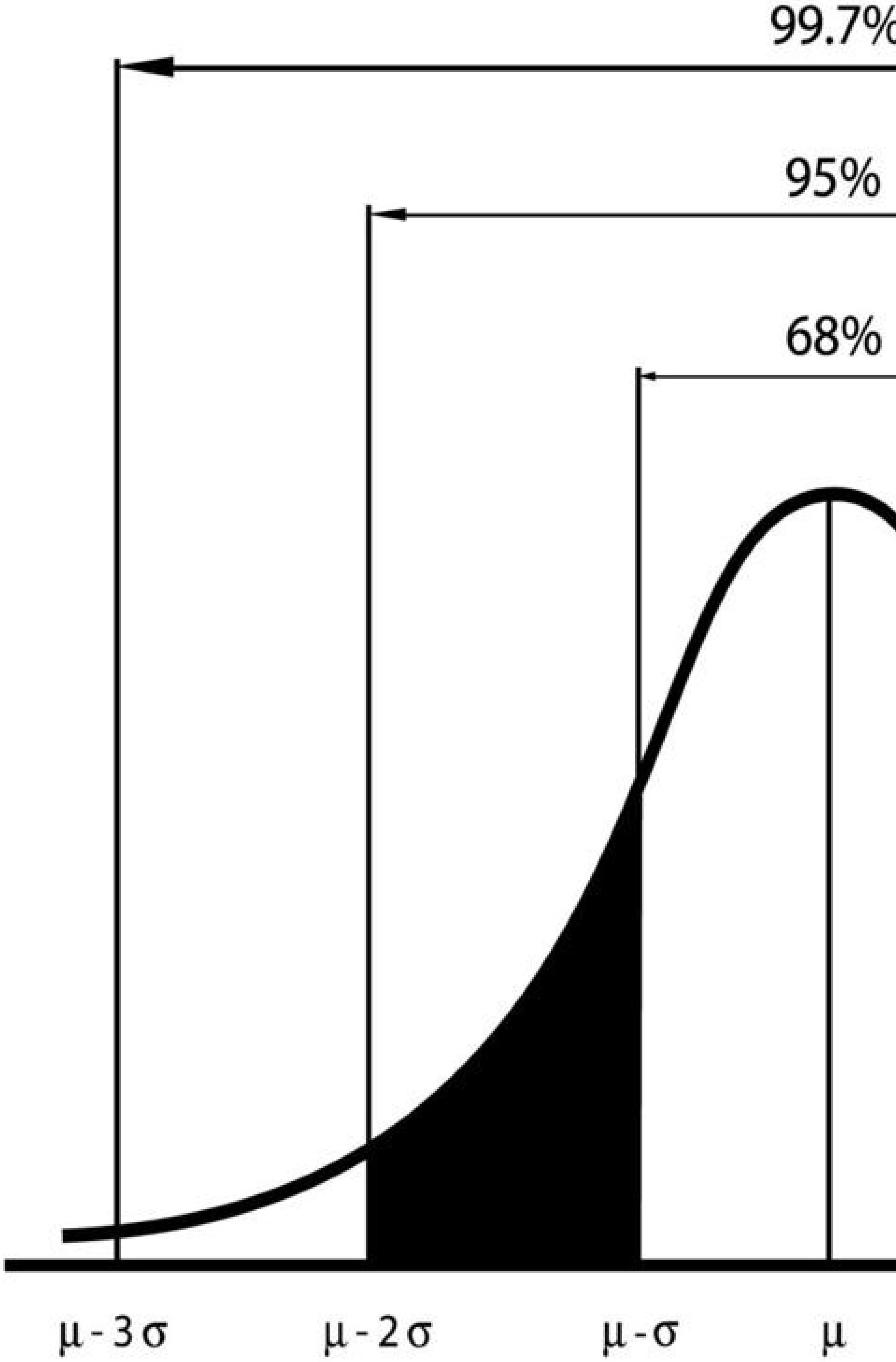
Standard Deviation Distribution ?

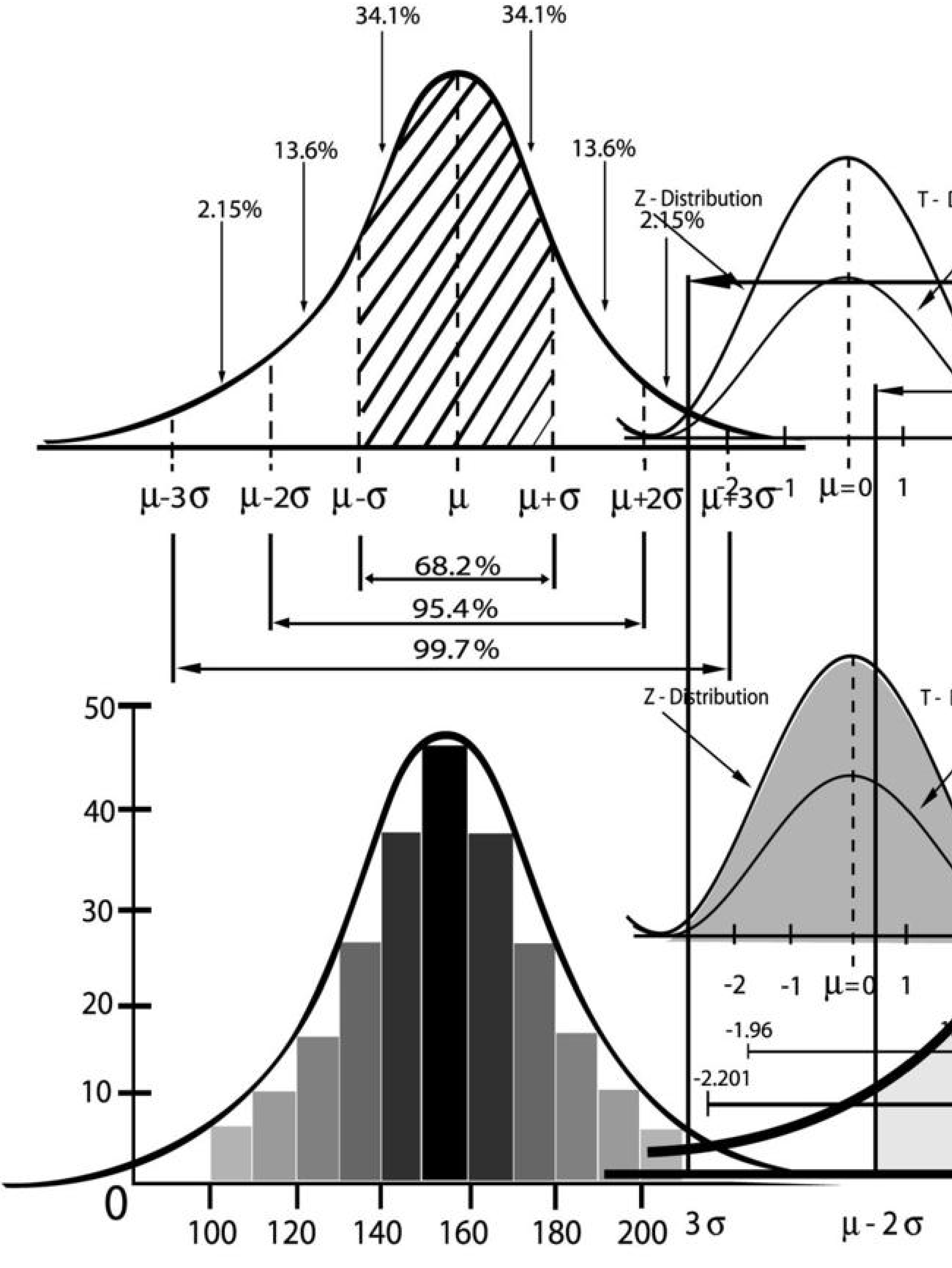
- **68%** of data ? within **±1 SD**
- **95%** of data ? within **±2 SD**

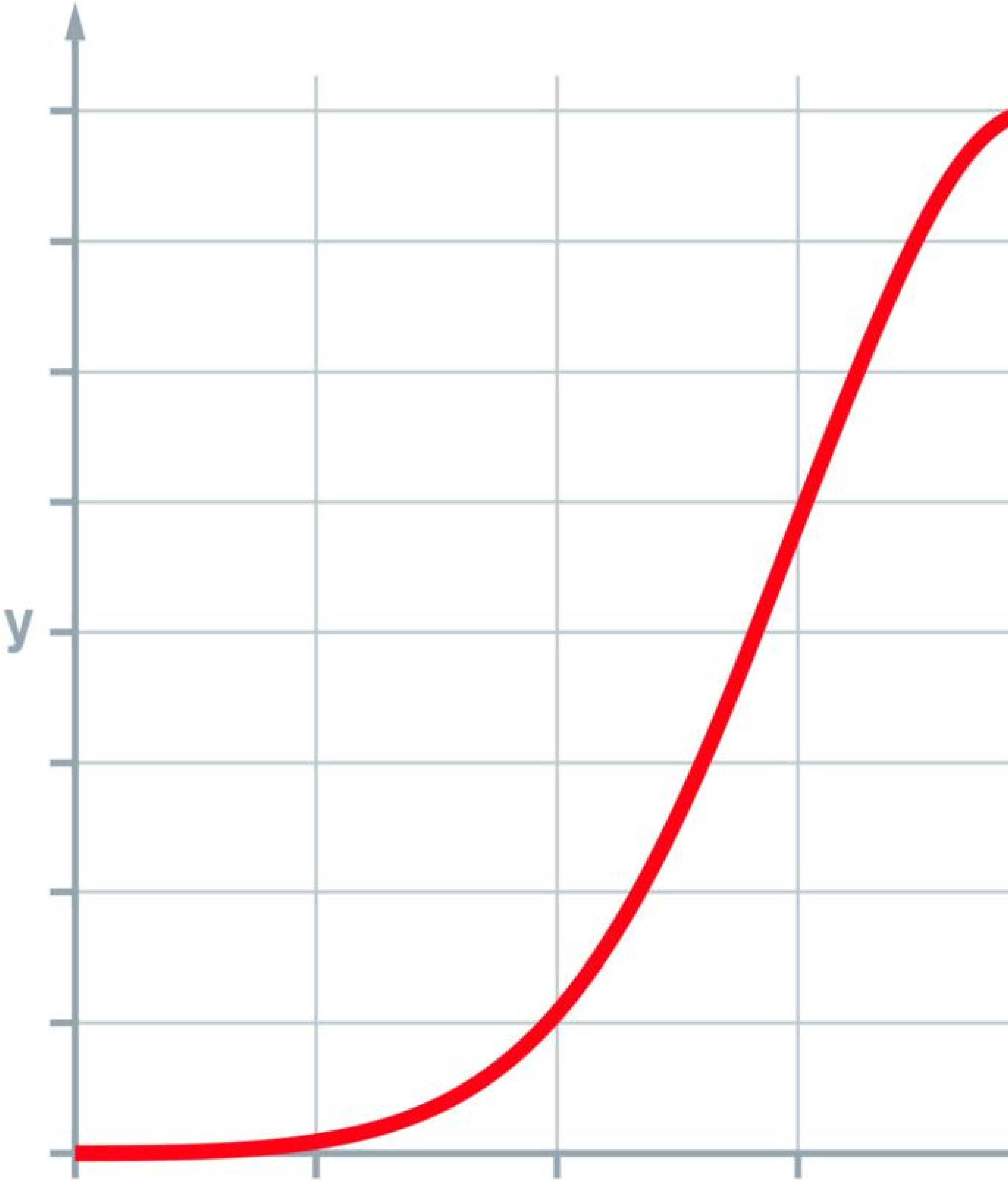
- **99.7%** of data ? within **±3 SD**

? This is called the **Empirical Rule (68–95–99.7 rule)**

Diagram (Bell-shaped Curve with SD Markings) ?


99.7%
95%
68%
μ - 3σ
μ - 2σ
μ - σ
μ


34.1%
34.1%
13.6%
13.6%
2.15%
Z - Distribution
2.15%
μ-3σ
μ-2σ
μ-σ
μ
μ+σ
μ+2σ
μ+3σ
-2
-1
μ=0
1
68.2%
95.4%
99.7%
Z - Distribution
50
40
30
20
10
0
100
120
140
160
180
200
3σ
μ - 2σ
-1.96
-2.201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Interpretation ?

- **Narrow curve ? Small SD ? Less variability**
- **Wide curve ? Large SD ? More variability**
- Majority of values cluster around the **mean**

Clinical / Epidemiological Relevance

- Biological variables:
    - Height
    - Weight
    - Blood pressure
- Used in:
    - **Reference ranges**
    - **Z-score calculations**
    - **Statistical tests**

Important Exam Points ?

- Bell-shaped curve
- Mean = Median = Mode
- 68–95–99.7 rule (VERY FREQUENT MCQ)

Short Note (Revision)

- Normal distribution ? symmetrical
- Mean = Median = Mode
- 68% ? ±1 SD
- 95% ? ±2 SD
- 99.7% ? ±3 SD

Uses of Dispersion

Core Uses ?

- **Measure reliability of data**

    - Less dispersion ? **more reliable data**
    - More dispersion ? **less reliable**

- **Compare datasets**
    - Helps compare variability between two or more groups
    - Example: Using **SD or CV** to compare consistency

- **Understand variability**
    - Shows how much data values **differ from the average**
    - Helps identify **spread and distribution pattern**

Easy Example

- Dataset A ? SD = 5 (less spread)
- Dataset B ? SD = 20 (more spread)
  ? Dataset A is **more consistent**

Public Health Applications ?

- **Epidemiological studies**
    - Assess variation in:
        - Disease occurrence
        - Risk factors

- **Research interpretation**
    - Helps interpret:
        - Study results
        - Clinical trial outcomes

Clinical Example

- Blood pressure readings:
    - Low SD ? consistent readings
    - High SD ? fluctuating readings

Important Exam Point ?

- Dispersion helps in:
    - **Reliability**
    - **Comparison**
    - **Understanding variability**

Short Note (Revision)

- Measures spread
- Helps compare datasets
- Indicates consistency
- Useful in epidemiology & research